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The effect of T-stress on the brittle fracture under mixed mode loading

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Abstract

Over the recent years, an increasing attention has been devoted to study the effect of higher order terms of Williams' series expansion in the fracture mechanics problems. It has been shown that the stress and strain fields around the crack tip are influenced considerably by the constant term, *T*-stress, and considering the effect of this term leads to better results. Several fracture criteria have been developed to describe brittle failure in linear elastic bodies, when subjected to mixed mode (I/II) loading. In this paper, the conventional fracture criteria, namely maximum tangential stress criterion, minimum strain energy density criterion, Maximum dilatational strain energy criterion, and maximum triaxial stress criterion are modified to take into account the effect of *T*-stress. A comparison between the effect of T-stress on the different fracture criteria has been made. The results from various criteria are then compared with the earlier experimental results for PMMA subjected to mixed mode loading.

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Keywords: Fracture mechanics; Mixed mode I/II; Stress intensity factors; T-stress; Fracture initiation

1. Introduction

Brittle fracture is a major mode of failure in components containing cracks. Crack growth in brittle materials often takes place very fast and with serious consequences. Therefore, it is important to define an appropriate procedure for predicting the onset of brittle fracture in cracked specimens. Many researchers in the field of fracture mechanics have shown an interest in studying the mixed mode fracture of brittle materials and several criteria have been proposed for predicting the mixed mode fracture [1-9].

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Over the recent years, an increasing attention has been devoted to study the effect of higher order terms of Williams' series expansion on the initiation of mixed mode fracture. Many researchers [e.g. 10, 11] showed that there is a difference between mixed mode fracture based on conventional and the experimental results. The main reason for this difference has been attributed to the presence of *T*-stress, the constant stress parallel to the crack. Previous studies show that the stress intensity factors K_I and K_{II} together with the *T*-stress provide more reliable predictions for mixed mode fracture. The *T*-stress effects on fracture toughness, crack growth rate and crack growth path stability have been investigated by many researchers, e.g. Larsson and Carlsson [12], Rice [13], Ayatollahi et al. [14] and Ayatollahi and Sedighiani [15]. The results presented by Bilby et al. [16] point to the dependence of crack tip constraint on the *T*-stress. Considering the relationship between the *T*-stress and the fracture resistance of materials, two-parameter models for mode I cracks and three-parameter models for mixed mode cracks is required to be established for a reliable fracture prediction in cracked components.

In this paper, the conventional fracture criteria, namely maximum tangential stress (MTS) criterion [1], minimum strain energy density criterion (S-criterion) [2, 3], maximum triaxial stress criterion (M-criterion) [4-6], and maximum dilatational strain energy criterion (T-criterion) [7] has been reviewed to take into account the effect of *T*-stress. The results from various criteria are then compared with the earlier experimental results for PMMA subjected to mixed mode conditions.

2. Analysis

Mixed mode elastic stresses around the crack tip can be written as a set of series expansions as

$$
\begin{aligned}
\sigma_x &= \frac{1}{\sqrt{2\pi r}}\left(K_I\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)-K_{II}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\right)+T\\
\sigma_y &= \frac{1}{\sqrt{2\pi r}}\left(K_I\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+K_{II}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\\
\tau_{xy} &= \frac{1}{\sqrt{2\pi r}}\left(K_I\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}+K_{II}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right)\\
\sigma_z &= 0 \qquad \text{for plane stress}\\
\sigma_z &= \nu(\sigma_x+\sigma_y) \qquad \text{for plane strain}
\end{aligned}
\tag{1}
$$

where r and θ are the polar coordinates with the origin located at the crack tip and υ is Poisson's ratio. σ_x, σ_y, τ_{xy}, and σ_z are Cartesian stresses.

The conventional criteria are often proposed on the basis of singular term of elastic stresses at a constant distance from the crack tip. This distance, however, has no effect on the prediction of mixed mode fracture by these criteria, when only the singular term in the Williams' series expansion is used. Considering the effect of *T*-stress in the conventional criteria requires five parameters K_I, K_{II}, T, K_{IC} and r_c for estimating the mixed mode I/II fracture resistance. K_I, K_{II} and T depend on the geometry and loading conditions in the test specimen whereas the critical distance r_c and fracture toughness K_{IC} are considered to be material properties. Therefore, for considering the effect of *T*-stress in the conventional criteria, an appropriate and reasonable value should be taken for r_c.

The critical distance r_c can be considered as a parameter that describes the length of damage zone developing around the crack tip. This damage zone is generated because of high stress/strain concentration in the vicinity of the crack tip. As a simple model for estimating the size of r_c, the following equation has been suggested by Taylor et al. [17]:

$$
r_c = \frac{1}{2\pi}\left(\frac{K_{IC}}{\sigma_t}\right)^2 \tag{2}
$$

where σ_t is the material tensile strength. In the following the aforementioned criteria for mixed mode fracture are described and then the effect of *T*-stress on them is investigated.

2.1. MTS criterion

Erdogan and Sih [1] proposed the Maximum tangential stress (MTS) criterion. This criterion states that the onset of crack extension takes place in the direction of maximum tangential stress along a constant radius around the crack tip. MTS criterion can be expressed mathematically as follows:

$$\frac{\partial \sigma_\theta}{\partial \theta} = 0, \qquad \frac{\partial^2 \sigma_\theta}{\partial \theta^2} < 0 \tag{3}$$

Smith et al. [9] later presented a modified MTS criterion. The modified criterion takes into account the effects of both the singular terms and the *T* term in the tangential stress around the crack tip.

2.2. S-criterion

The S-criterion, formulated by Sih [2, 3] states that the direction of fracture initiation coincides with the direction of minimum strain energy density along a constant radius around the crack tip. It is stated as

$$\frac{\partial S}{\partial \theta} = 0, \qquad \frac{\partial^2 S}{\partial \theta^2} > 0 \tag{4}$$

The strain energy density factor S can be expressed in terms of the stress components as follow:

$$S = \frac{1}{2G}\left[\frac{\kappa+1}{8}\left(\sigma_{rr} + \sigma_{\theta\theta}\right)^2 - \sigma_{rr}\sigma_{\theta\theta} + \tau_{r\theta}^2\right] \tag{5}$$

where *G* is the modulus of rigidity, and κ is (3-ν)/(1+ν) for plane stress and (3-4ν) and for plane strain.

2.3. T-criterion

This criterion states that when the dilatational strain energy density at a point in the vicinity of crack tip reaches a critical value, the crack starts to propagate. This criterion uses elastic-plastic boundary calculated by Von Mises yield criterion to define the radius of critical distance and can be expressed as:

$$\frac{\partial T_\upsilon}{\partial \theta} = 0, \qquad \frac{\partial^2 T_\upsilon}{\partial \theta^2} < 0 \tag{6}$$

where T_v is the dilatational strain energy.

2.4. M-criterion

The M-criterion, presented by Kong [7] states that the crack initiates in the direction of the maximum triaxial stress ratio (M) along a constant radius around the crack tip. It can be stated mathematically as

$$\frac{\partial M}{\partial \theta} = 0, \qquad \frac{\partial^2 M}{\partial \theta^2} < 0 \tag{7}$$

where the stress triaxiality ratio *M* is σ_H/σ_{eq}. σ_H is the hydrostatic stress and σ_{eq} is the equivalent stress.

3. Results and discussion

In this section the results for aforementioned criteria are presented. To normalize the contribution of the *T*-stress relative to the stress intensity factors K_I and K_{II}, a dimensionless parameter called the biaxiality ratio, *B*, has been suggested by Leevers and Radon [18]

$$B = \frac{T\sqrt{\pi a}}{\sqrt{K_I^2 + K_{II}^2}} \tag{8}$$

As mentioned earlier, estimating the mixed mode I/II fracture in the presence of *T*-stress is depending on defining an appropriate value for critical radius r_c. In this paper, $\alpha=(2r_c/a)^{0.5}$ are used to normalize the effect of r_c on the mixed mode fracture. The ratio of mode I and mode II stress intensity factors is often presented by a dimensionless parameter called the mode mixity factor M^e which is written as

$$M^e = \frac{2}{\pi}\tan^{-1}\left(\frac{K_I}{K_{II}}\right) \tag{9}$$

The value of M^e is unity for pure mode I and zero for pure mode II. Fig. 1 shows the variation of crack initiation angle versus M^e for different values of $B\alpha$ calculated from four different criteria. It is seen that the curves exhibit more deviations from the conventional criteria curves ($B\alpha$=0) for materials having larger values of $B\alpha$. The results calculated based on the modified criteria reveal that a positive *T* increases the initial angle of crack propagation whereas a negative *T* reduces the fracture initiation angle. It is clear from these figures for $B\alpha$=0.4 the fracture angle for mode I does not coincide with the plane of the crack, however, there is a considerable difference between the predicted results by S-criterion with other criteria.

Mixed mode fracture loci based on the modified MTS, S, and T criteria for different values of $B\alpha$ are given in the Fig. 2. The results suggest that the mixed mode fracture toughness of a cracked specimen depend considerably on the magnitude and the sign of *T*-stress. It is obvious that the mixed mode fracture toughness increases by a negative $B\alpha$ and decreases by a positive Bα. For the tangential Stress and under Pure more I loading, the *T*-term will be vanished in the stress expansion. Therefore, based on modified MTS criterion the *T*-stress has no effect on the mode I fracture initiation. While, for modified S and T criteria *T*-stress would contribute to the equations under mode I loading and consequently the fracture behaviour will be affected by the *T*. These results reveal that based on these two criteria the load bearing capacity of the cracked components is considerably depending on the geometry and loading conditions.

Pervious studies [9-11] reveal a difference between mixed mode fracture predicted by conventional criteria with the experimental results. In this paper four conventional criteria were modified to take into account the effect of *T*-stress. Here, the results calculated from the modified criteria are compared to the previous experimental results for the angled center-crack specimen.

For an angled center-crack specimen the biaxiality ratio is positive for 0≤β≤45 while it is negative when 45≤β≤90 (β is the crack inclination angle). Therefore, it is predictable that for 0≤β≤45 the fracture angle and fracture toughness would be respectively higher and lower then those predicted by conventional criteria. The opposite is expected for 45°≤β≤90°. Fig. 3(a) shows the experimental results of fracture initiation angle presented by Ueda et al. [11] together with curves obtained from the modified criteria as a function of M^e. It is seen that the modified criteria provide very good estimates for the fracture initiation angles obtained from the experiments. The results calculated from modified MTS and S criteria are very close and lower than the average value of the experimental results. While, the results predicted by T-criterion are a little more than the average value of the experimental results.

The experimental results of fracture toughness presented by Williams and Ewing [10] and Ueda et al. [11] together with results calculated for modified criteria are shown as a function of M^e in Fig. 3(b). The results for modified criteria are based on α=0.2, as suggested by [10]. Again, the results calculated from modified MTS and S criteria are very close with a little difference under mixed mode condition. The T-criterion predicts higher values for mixed mode fracture toughness of the specimen. A comparison between experimental results and three modified criteria reveals that for mode I dominated loading the modified MTS and S criteria produce a better result, whereas under mode II dominated loading the results calculated by T-criteria are in a better agreement with the experimental results.

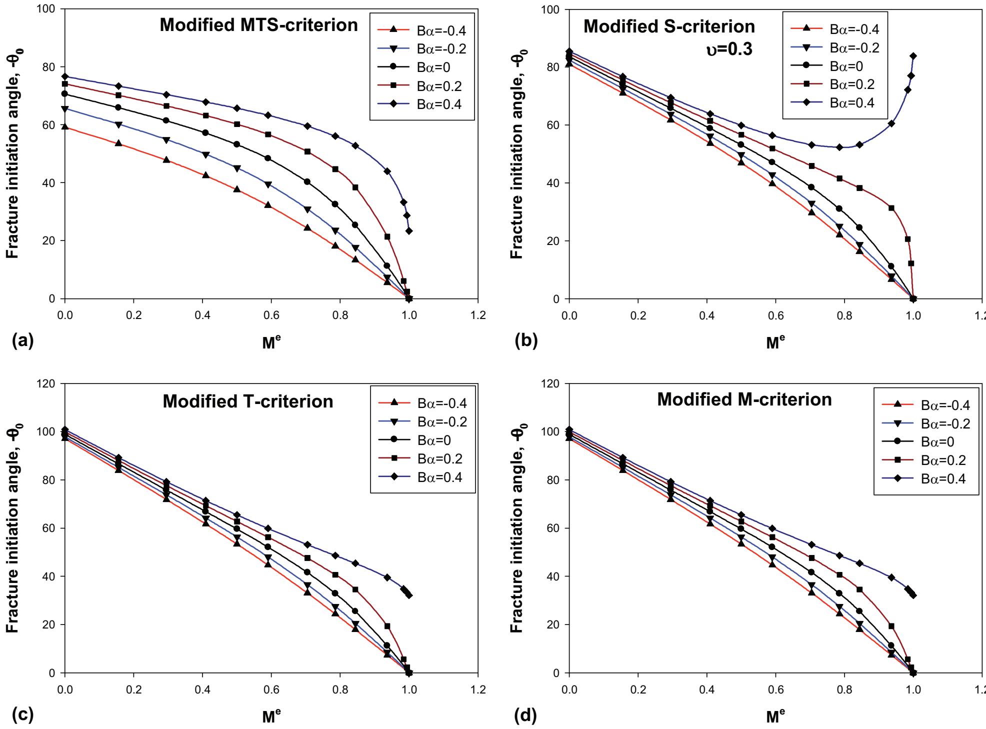

Fig. 1. The effect of *T*-stress on the fracture initiation angle; (a) modified MTS; (b) modified S; (c) modified T; (d) modified M.

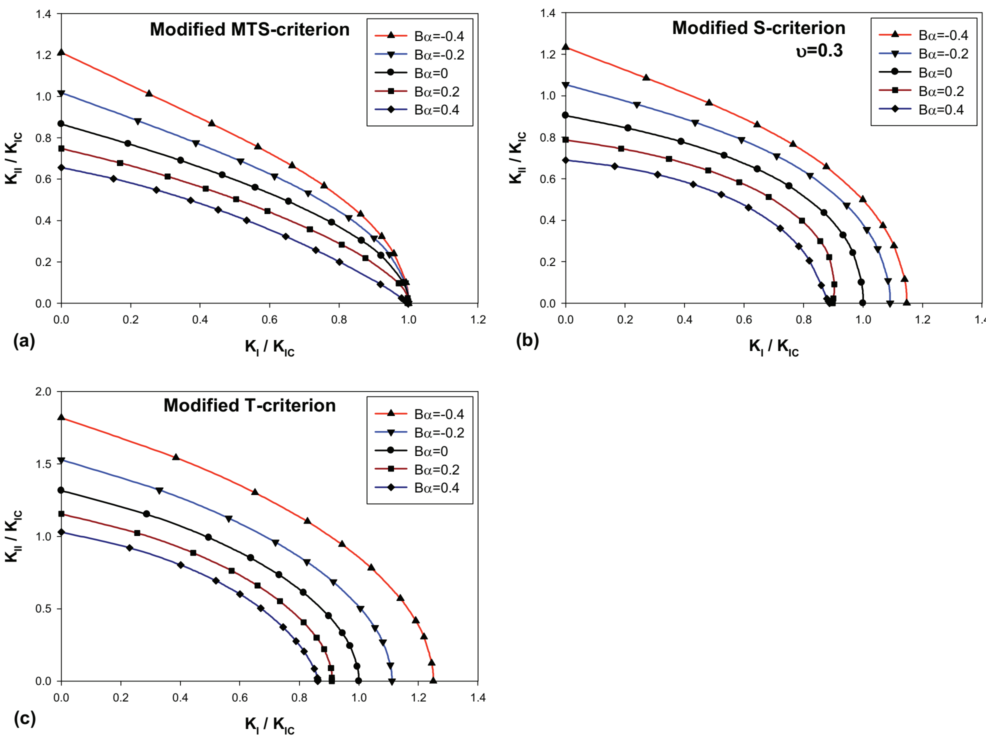

Fig. 2. The effect of T-stress on the mixed mode fracture loci; (a) modified MTS; (b) modified S; (c) modified T.

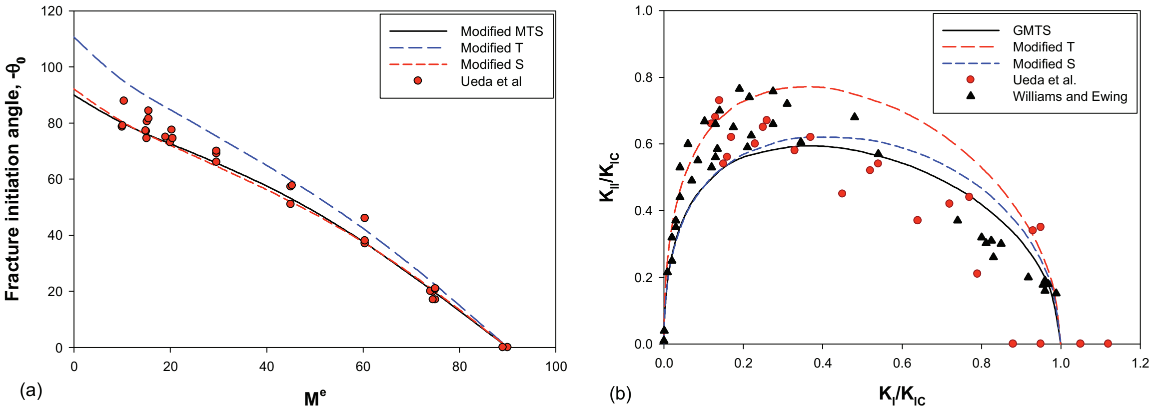

Fig. 3. (a) Fracture initiation angle of angled center-crack specimen; (b) Fracture toughness of angled center-crack specimen.

4. Conclusion

In this paper four conventional criteria for predicting brittle fracture in linear elastic materials were modified to take into account the effect of *T*-stress. It was shown that *T*-stress has a significant influence on the both fracture initiation angle and mixed mode fracture resistance. Based on the modified S and T criteria the mode I fracture toughness is also influenced by the *T*-stress, whereas, based on modified MTS criterion *T*-stress has no effect on the mode I fracture resistance.

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